LU

$$\text{for } k = 1 : n - 1$$
$$\quad \text{for } i = k + 1 : n$$
$$\quad\quad m_{i,k} = a_{i,k}^{(k)} / a_{k,k}^{(k)}$$
$$\quad\quad \text{for } j = k + 1 : n$$
$$\quad\quad\quad a_{i,j}^{(k+1)} = a_{i,j}^{(k)} - m_{i,k} a_{k,j}^{(k)}$$
$$\quad\quad \text{end}$$
$$\quad \text{end}$$
$$\text{end}$$

$$L = \begin{bmatrix} 1 & & & & \\ m_{2,1} & 1 & & & \\ m_{3,1} & m_{3,2} & 1 & & \\ \ldots & & & & \\ m_{n,1} & m_{n,2} & m_{n,3} & \ldots & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} a_{1,1}^{(1)} & a_{1,2}^{(1)} & a_{1,3}^{(1)} & \ldots & a_{1,n}^{(1)} \\ 0 & a_{2,2}^{(2)} & a_{2,3}^{(2)} & \ldots & a_{2,n}^{(2)} \\ 0 & 0 & a_{3,3}^{(3)} & \ldots & a_{3,n}^{(3)} \\ \ldots & & & & \\ & & & & a_{n,n}^{(n)} \end{bmatrix}$$

Metodi Iterativi

$$A\mathbf{x} = \mathbf{b}$$

▶ Test d'arresto basato sul residuo $\mathbf{r}^{(k)} := \mathbf{b} - A\mathbf{x}^{(k)}$

$$\mathbf{r}^{(k)} = \mathbf{b} - A\mathbf{x}^{(k)} = A(\mathbf{x} - \mathbf{x}^{(k)})$$

$$\mathbf{e}^{(k)} = \mathbf{x} - \mathbf{x}^{(k)} = A^{-1}\mathbf{r}^{(k)} \qquad \|\mathbf{e}^{(k)}\| \leq \|A^{-1}\|\|\mathbf{r}^{(k)}\|$$

$$\|\mathbf{b}\| \leq \|A\|\|\mathbf{x}\| \qquad \|\mathbf{x}\| \geq \frac{\|\mathbf{b}\|}{\|A\|}$$

$$\frac{\|\mathbf{e}^{(k)}\|}{\|\mathbf{x}\|} \leq \|A\|\|A^{-1}\|\frac{\|\mathbf{r}^{(k)}\|}{\|\mathbf{b}\|}$$

Metodi Iterativi

$$A\mathbf{x} = \mathbf{b} \qquad \mathbf{x} = B\mathbf{x} + \mathbf{c} \qquad \mathbf{x}^{(k+1)} = B\mathbf{x}^{(k)} + \mathbf{c}$$

▶ Test basato sull'incremento $\mathbf{inc}^{(k)} := \mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}$

$$
\begin{aligned}
\mathbf{e}^{(k)} = \mathbf{x} - \mathbf{x}^{(k)} &= \mathbf{x} - \mathbf{x}^{(k+1)} + \mathbf{x}^{(k+1)} - \mathbf{x}^{(k)} \\
&= B(\mathbf{x} - \mathbf{x}^{(k)}) + (\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)})
\end{aligned}
$$

$$\mathbf{e}^{(k)} = B\mathbf{e}^{(k)} + (\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}).$$

Se $\|B\| < 1$

$$\|\mathbf{e}^{(k)}\| = \frac{1}{1 - \|B\|}\|\mathbf{inc}^{(k)}\|$$