Course in
Numerical Approximation of Partial Differential Equations:
The Finite Element Method

Instructor: Ana Alonso Rodríguez

Contents:

- Introduction:

  Preliminary notation and function spaces. Some results about Sobolev spaces. Variational formulation of a model problem. The Lax Milgram lemma. Galerkin method.

- The construction of finite element spaces:

  Triangulation. Piecewise polynomial subspaces. The interpolation operator.

- Finite element approximation of elliptic problems:

  Variational form of elliptic boundary value problems. Regularity of solution. Error estimates.

- Remarks on implementation and algorithmic aspects:

  Representation of the triangulation. Computation of the stiffness matrix. The condition number of the stiffness matrix.

- Numerical solution of linear systems:

  Direct and iterative methods. The conjugate gradient and related methods. Preconditioning.

- Steady advection-diffusion problems:

  Mathematical formulation. A one dimensional example. Stabilization Methods.

- The Stokes problem:

  Mathematical formulation. Mixed methods. Iterative techniques for mixed methods.

- Finite element approximation of parabolic problems:

  Initial-boundary value problems and weak formulation. Semi-discrete approximation. Time-advancing by finite differences.

References:

- A. Quarteroni and A. Valli, *Numerical Approximation of Partial Differential Equations*, Springer, 1997 (2nd printing).

- S. C. Brenner and L. R. Scott, *The mathematical Theory of Finite Element Methods*, Springer, 1994.

Duration: 36 hours. 4 hours a week from the begining of March to the end of May.